2006

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ABSTRACT

This paper develops a semi-empirical expression for heat transfer in round tube plain fin heat exchangers that have relatively large fin pitch. Empirical heat transfer correlations currently available in the public domain are necessarily limited in their range of applicability, and their functional forms are generally not designed to converge to well-known physical results, for example when parameters such as fin or tube pitch become large as in the case of frost-tolerant evaporators designed for refrigeration applications. The approach taken here begins with a simple superposition of a fin array and tube bank, and then adds empirically-based terms to account explicitly for the streamwise vortices that form at fin-tube junctions and the spanwise von Karman vortex streets shed from the tubes. Comparisons with available data show good agreement for heat exchangers having large fin pitch, where these effects are dominant. Developing flow effects are also treated explicitly. The resulting expression provides a framework for modifying the purely empirical correlations developed for air conditioning applications, and bridging the gap to sparser data available for refrigeration coils.

1. INTRODUCTION

The purely empirical correlations currently used to characterize air-side heat transfer and pressure drop in round-tube heat exchangers are based on data taken over several decades by several investigators, using samples of production and prototype heat exchangers that reflected the technologies that were dominant at the time. As new technologies enable the use of smaller tube diameters and thinner fins in air conditioning, or new applications such as heat pumps, such correlations cannot be used to identify or analyze potentially optimal configurations that may lie outside the historically-defined parameter space. Unless the functional form of a correlation accurately reflects the physics of the flow, the Buckingham pi theorem cannot produce nondimensional variables that can be used to extrapolate beyond the dimensional limits of the underlying data set.

This paper takes a physically-based approach to developing a heat transfer correlation that deals only with plain fins (Wang et al., 2000), but the motivation is similar (and data sets sparser) for those with slit or louvered fins. The goal is to provide a functional form capable of being extrapolated to such well-known limiting conditions as fin arrays or tube banks, explicitly representing physical phenomena that tend to dominate heat transfer when fin pitch is large: e.g. developing flow and vortices shed from tubes and tube-fin junctions. A broader goal is to establish a mathematical framework that is also capable of representing the very different physical phenomena observed at small fin pitch, as in a/c evaporators and condensers.

Preliminary analyses using a system simulation model to optimize condenser geometry for a typical 10.5 kW split a/c system at the ARI-A design condition illustrated the inherent limitations of published correlations. The objective was to maximize system efficiency subject to a constraint on heat exchanger mass. For one-row heat exchangers the optimization algorithm was constrained by the lower bound on fin pitch, and for multi-row heat exchangers it was quickly constrained by upper bounds on both tube and fin pitches. Given the growing need to design heat pumps that can operate efficiently under frosting conditions, and the inability to extrapolate published correlations to larger tube and fin pitches, the analytical focus shifted to characterizing the physical phenomena that dominate heat transfer in such coils.

The analytical and semi-empirical solutions developed by Stefan (1959) and Zhukauskas (1976) for fin arrays and tube banks, respectively, serve as a starting point for developing a mathematical expression that approaches those limits as tube or fin pitch increases. Ichimyia (1988) measured temperature profiles at constant heat flux on a duct wall created by root vortices formed at the fin-tube junction, and observed heat transfer enhancements far greater than those observed by Saboya and Sparrow (1974,1976) in naphthalene sublimation experiments at small (<3 mm) fin pitch where the fin interactions tend to quickly dissipate both spanwise and streamwise vorticity, leaving a long stagnant wake downstream of the tubes. Kawamura et al. showed that the root vortices have little effect on tube surface heat transfer, simplifying the task of incorporating interaction effects into a physically-based formulation.

There is relatively little published data for heat exchangers having fin pitch of the same order of magnitude as the tube diameter. Kim and Kim (2005a) conducted experiments on a family of prototype heat exchangers having 10 mm tubes with identical pitch, with fin pitch varying between 7 and 15 mm, and tube rows from 1 to 4. As other experiments have shown, those with staggered tubes outperformed the inline prototypes, so this analysis focuses only on the former. Subsequent to publication the authors have developed separate curve fits for their staggered tube data, and separate correlations for their one-row and multi-row prototypes, reducing their rms error to ~1% (Kim and Kim, 2005b). Those curve fits are used in this paper as surrogates for the underlying unpublished data, to test our semi-empirical model.

Granryd (1965) accumulated a much larger set of data spanning a much larger part of the parameter space where fin pitch exceeds 3 mm. Again, the data were not published so only the correlations are available as a basis for evaluating our physically-based approach. Unfortunately due to Granryd's focus on refrigeration coils having relatively large tube diameters and face velocities, the data sets do not overlap Kim's. Moreover only a small fraction of the data had fin pitch greater than half the tube diameter. Granryd's semi-empirical approach, however, is consistent with ours, as it estimates empirical parameters to capture the effects of small fin pitch while allowing extrapolation to tube banks and fin arrays.

The following sections describe the first steps towards development of a single correlation for air-side heat transfer in single-and multi-row heat exchangers that can cover the entire parameter space with one continuous function. Such a formulation is required for compatibility with commonly used optimization algorithms that may be applied to heat exchanger and system design. The approach described below begins with a simple superposition of a fin array and tube bank, and then draws upon various experimental investigations to add empirically-based terms to account explicitly for the streamwise vortices that form at fin-tube junctions and the spanwise von Karman vortex streets shed from the tubes.

2. Methodology

Equation (1) is Stephan's analytical expression for simultaneously developing flow in a 2-D duct with an isothermal boundary; it has been found to fit numerical solutions within 3% (Kakac, *et al.* 1987). Equation (2) for staggered tube banks quantifies the heat transfer enhancement experienced by downstream tube rows as a result of the von Karman vortex streets shed by the first and subsequent rows.

$$Nu_{2D,Stephan} = 7.55 + \frac{0.024 \cdot x^{*-1.14}}{1 + 0.0358 \cdot \Pr^{0.17} \cdot x^{*-0.64}} \qquad \text{where} \qquad x^* = \frac{depth}{D_h \cdot \mathrm{Re}_{Dh} \cdot \Pr} \tag{1}$$

Superposition of such flow fields requires an assumption of linearity that could only be justified at large fin or tube pitch where interactions could be neglected. The analysis presented in the next section begins with this simple case, and then proceeds to model a subset of interaction effects, perhaps ironically by employing a superposition approach.

This basic assumption follows from the fact that the large-scale flow fields produced by vortices are routinely analyzed using potential flow theory, as the most significant viscous effects are localized in the vortex core. The Zhukauskas correlation quantifies the effects of the vortex streets shed from tubes as their transverse velocity components enhance heat transfer on the tube surfaces that lie downwind. It also quantifies the viscosity-driven decay of those vortex streets as they are convected downstream through the tube bank. Therefore we assume *for* *large fin pitch* that this heat transfer enhancement mechanism operates in a simple additive manner as the vortex-induced components of velocity combine nondestructively with the streamwise laminar flow field between two widely-spaced parallel plates. At small fin pitch, however, the viscous core of the vortices is dissipated quickly due to the proximity of the tube walls, so the large-scale flow structures are unable to form and be carried downwind where they can enhance heat transfer on tube and fin surfaces.

Empirical data are also available to characterize the heat transfer-enhancing effects of the horseshoe vortices formed at the root of a cylinder protruding from a wall, normal to the air flow direction. In inviscid potential flow, the two streamwise vortices would begin to diverge from one another under the influence of the other's far-field velocity vector. For a similar reason the vortices would quickly move away from the wall, as if it were pushed away by a mirror image vortex on the opposite side of the wall. With the vortices thus removed from the dissipating effects of nearby surfaces, their large-scale velocity fields are carried downstream and their spanwise velocity component is available to enhance heat transfer on the duct walls. Similarly, their velocity components normal to the walls can bring fresh air into the thermal boundary layer that would otherwise be thickened as the laminar flow proceeded through the duct.

Ichimiya's (1988) experiments using acrylic cylinders of various diameters located in fully developed laminar flow through duct produced valuable insights into the nature of the heat transfer enhancements associated with root vortices. Even in laminar flow, most of the enhancement occurred on the wall near the root of the cylinder where the viscous core of the vortices were close to the surface and the transverse velocity components were relatively large. However there were also heat transfer enhancements of substantial magnitude extending more than 5 tube diameters downwind, as the large-scale flow field of the streamwise vortex was carried downwind. When fin pitch is large the viscous effects (scouring near the tube root, and rate of decay downstream) are represented by a simple additive empirical curve fit of heat transfer enhancement factor expressed by Ichimiya as a function of downstream distance and enhanced area, normalized with respect to tube diameter and linearly interpolated between Reynolds numbers of 1000 and 2000 based on tube diameter.

In any event for most heat exchanger geometries tube area is only a small fraction of the total, diminishing the effect of their area-weighted contribution. The next section describes in detail how the semi-empirical correlation is constructed from the analytical and empirical results described above.

2. Semi-empirical correlation for large fin pitch

The basic framework of the correlation involves a simple area-weighted averaging of the fin and tube heat transfer coefficients. Since the effect of the fins on the tubes is neglected, all interaction effects are embodied in the multiplicative enhancement factor applied to the fin heat transfer coefficient.

$$h_{air} = \frac{EF_{fins} \cdot h_{fins} \cdot A_{fins} + h_{tubes} \cdot A_{tubes}}{A_{fins} + A_{tubes}} \qquad (2)$$

To account for local variations of the heat transfer coefficients and the enhancement mechanisms, the fin is divided into N_{rows} + 1 segments as illustrated in Figure 1 for a 4-row heat exchanger.

Next, these local results are then aggregated as shown in Equation (3). In fin segment A_1 the heat transfer

$$EF_{fins} = \frac{\sum_{i=1}^{N_{rows}+1} EF_i \cdot h_{loc,i} \cdot A_i}{h_{fin} \cdot A_{fin}} \qquad (3)$$

$h_{loc,i}$ is given by Eq. for a 2-D duct and the local velocity $V_{channel}$, which is simply the face velocity adjusted for the blockage effect of the fin thickness. By definition, $EF_1 \equiv 1$ for area A_1 because all fin-tube interaction effects are assumed to occur downstream of the first tube row.

For all other segments starting with area A_2, the local enhancement factor is computed as follows:

$$EF_i = (EF_{Ichi,i} - 1)\frac{A_{enh,i}}{A_i} + EF_{Z,i} \qquad (4)$$

where EF_{Ichi} is from Ichimiya and applies only to the $A_{enh,i}$ enhanced by the horseshoe vortex extending downwind of a tube in row i, and $EF_{Z,i}$ is local enhancement factor observed by Zhukauskas on the surface of a tube in row i, representing the cumulative effect of all the vortex streets shed from upstream tube rows. The following subsections treat 1-row and multi-row heat exchangers separately, because the vortex flow patterns differ significantly.

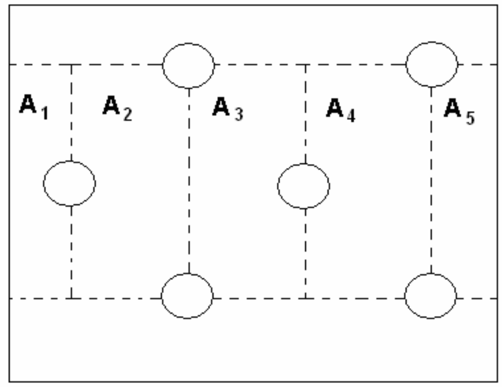

Figure 1. Area elements in fin section

2.1 One tube row

Figure 2 illustrates the shape of the wake region observed by Ichimiya (1988) on the wall downstream of a cylinder when fin pitch is large. The contours show how heat transfer is enhanced throughout the wake region when $Re_{Dc} \geq 1000$; greater enhancement was observed at larger Reynolds numbers. The wake region observed by Saboya and Sparrow at smaller fin pitch had generally the same shape, but enclosed a large stagnation region where heat transfer was severely degraded. In both cases the wake region extended more than 4 tube diameters downstream, beyond the trailing edge of any one-row heat exchanger. These observations form the basis for our decision to attribute the heat transfer enhancement to the transverse velocity component generated by the horseshoe vortex when fin pitch is large, and to conclude that small fin pitches confine the vortex to a small diameter along the shear layer separating the stagnation region from the free stream where $V=V_{max}$. Since the overall length and width of the wake region is the same, the same maximum velocity is assumed to prevail outside the wake when fin pitch is large. Accordingly the local enhancement factors in Eq. 4 are applied to local heat transfer coefficients in A_1 and A_2 that are calculated using $V_{channel}$ and V_{max}, respectively, as are the enhancement factors in Eq. 6.

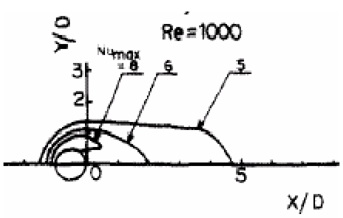

Figure 2. Ichimiya isotherms

The equation for the enhanced area, estimated from Fig. 2, is given by Eq. 5 and the

$$A_{enh} = 2 \cdot D^2 \cdot (1.8 + 1.35 \cdot (X / D)) \qquad X/D \leq 4 \tag{5}$$

$$A_{enh} = 2 \cdot D^2 \cdot (1.8 + 1.35 \cdot (X / D) + (X / D - 4)) \qquad 4 < X/D \leq 5$$

associated enhancement factor, which fits 9 data points from Ichimiya ±1%, is given by

$$EF_{Ichi} = A_1 \cdot (X / D) + A_2 \cdot (X / D)^{A_3} + B_1 \cdot \mathrm{Re}_{Dh} \tag{6}$$

where A_1 = 8.913e-3, A_2 = 1.122, A_3 = -8.692e-2, and B_1 = 8.658e-5. When applied to Eq. 4 for area A_2, the enhanced area equation is simply truncated at the trailing edge, and the enhancement factor in Eq. 6 is evaluated at X/D corresponding to the trailing edge.

Since Ichimiya's experiment with a single cylinder measured the combined effect of the spanwise vortex street as well as the streamwise horseshoe vortex that formed at the root, it is not necessary to treat these phenomena separately in the case of a one-row heat exchanger.

2.2 Multi-row heat exchangers

The same equations apply to the case of deeper heat exchangers, with a few adjustments to reflect the fact that air flowing at V_{max} between the tubes of row 1 is divided and diverted as it approaches the staggered tubes of row 2. The two streams converging to pass between the tubes of row 2 therefore confine the wake of tube 1 to area A_2, confining its heat transfer-enhancing root vortex to that small area instead of letting its effect to be distributed less intensively over a larger area. To quantify this effect, the term $EF_{Ichi}A_{enh}$ in Eq. 4 is evaluated by setting X/D=5, reflecting an admittedly crude but physically-based assumption that all the incremental heat transferred from the spanwise flow generated by the horseshoe vortex is confined to A_2 along with all the vorticity.

The second adjustment is rather straightforward, calculating the local heat transfer coefficients in Eq. 4 using V_{max} for all segments downstream of A_1. Since all downstream tube rows face V_{max} instead of $V_{channel}$ due to the blockage effects of both the tubes and the wakes, the downstream tube rows generate stronger spanwise and streamwise vortices, consistent with the findings that of both Zhukauskas and Saboya & Sparrow. Therefore V_{max} was used to calculate the enhancement factors starting with row 2. After the last row of tubes, the wake and its effects are truncated at the trailing edge, exactly as in the case of a one-row heat exchanger.

Table 1. Tube row multipliers

Number of Rows	Zhukauskas Average Multiplier
1	0.68
2	0.75
3	0.83
4	0.89
5	0.92
6	0.95
7	0.97
8	0.98
9	0.99
10	1

Finally the enhancement factor associated with the vortex streets shed by all tube rows is obtained directly from Zhukauskas' correlation for banks of staggered tube banks having N rows and transverse and longitudinal tube pitches S_T, S_L.

$$Nu_D = C \cdot C_2 \cdot \mathrm{Re}_{D,\max}{}^{0.6} \cdot \mathrm{Pr}^{0.36} \cdot \left(\frac{\mathrm{Pr}}{\mathrm{Pr}_{surface}} \right)^{0.25} \qquad (7)$$

In Eq. 7, C = $0.35(S_T/S_L)$ $N \le 2$ and 0.4 otherwise, and the average multipliers C_2 are given in Table 1. Recall, however, that Ichimyia's experiments quantified the near-field enhancement effect of the vortex on the fin area lying immediately downstream of the tube. However it is clear from Table 1 that the vortex streets shed by upstream tube rows continue to enhance heat transfer far downstream. From the average enhancement factors shown in the Table, it is straightforward to compute a local enhancement factor Z_i experienced by the tubes in each row i, relative to the heat transfer coefficient for tubes in row 1. In a 3-row heat exchanger, for example, the factor Z_i is applied to tube surfaces in row 3 and fin surfaces A_4.

4. Results and discussion

Heat transfer coefficients predicted by this physically-based approach compare well to empirical correlations at large fin pitch. Figure 3 compares the results of Eq. 2 for the heat exchanger prototype tested most extensively by Kim and Kim to their own empirical correlation, which can be viewed as a surrogate for data since its accuracy is
±1%. Figure 3 encompasses their entire range of face velocities and number of tube rows at 10 mm fin pitch. The largest discrepancy between the two correlations (5%) occurs at one row and high face velocity, and the rms error is 2%.

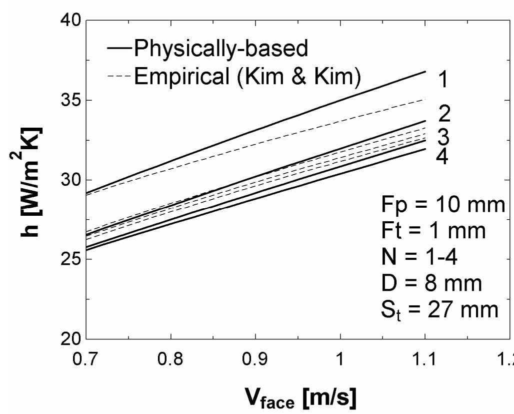

Figure 3. Comparison to Kim & Kim

While the data set at 10 mm fin pitch was dense, at the other fin pitches (7.5, 12.5, and 15 mm) it was limited to 1- and 2-row heat exchangers only. Nevertheless at 7.5 mm fin pitch, our semi-empirical correlation overestimates Kim & Kim's predicted heat transfer by a maximum of 10% (for one row and 1.1 m/s face velocity) and the rms error is 8%. On the other hand the agreement is much better 12.5 and 15 mm fin pitch, where rms errors are 2.7% and 3.6%, respectively. This trend is consistent with the physical assumptions underlying the semi-empirical approach: that closely-spaced fins dissipate the horseshoe vortices more rapidly, and at high fin pitch the velocity fields generated by the streamwise and spanwise vortex cores are additive, and therefore their heat transfer enhancements are additive.

Figure 4 is a comparison with Granryd's correlation, which again serves only as a surrogate for the underlying data because the detailed experimental results are unpublished. The geometric parameters for the 2- and 3-row cases are taken from Granryd's data set, but the 1-row prediction reflects data from prototypes having different fin and tube geometries. Again the agreement is good, even at this relatively small fin pitch where the vortices begin to dissipate more quickly. Most of Granryd's data covered tube diameters up to 35 mm and fin pitches down to 3 mm, and the majority of the heat exchangers had inline tube arrangements. Therefore the resulting empirical correlation is

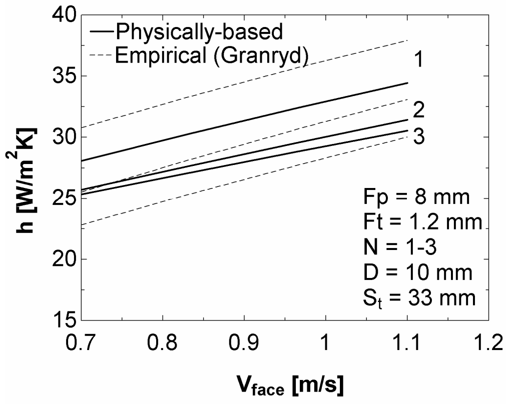

Figure 4. Comparison with Granryd

influenced by stagnation regions in many of the tube wakes, and wakes that extend to the next tube row. This may account for the relatively strong row dependence seen in Granryd's correlation, compared to the weaker dependence seen in Kim's staggered tube correlation and our semi-empirical results.

Due to the scarcity of published data, it is difficult to test many of the assumptions underlying the semi-empirical correlation presented in this paper. Therefore more detailed comparisons with available data are currently underway, aimed at confirming, refining or substantially revising the assumptions described here. For example the semi-empirical approach presented here deals with developing flow effects (thinner boundary layers near the leading edge) by applying the enhancement factors to local heat transfer coefficients on the fins. In combination with the assumption that the horseshoe vortices are concentrated in the local area immediately downstream of the tube, this may overestimate the enhancement effect. Although computationally more complex, it may be more accurate to assume that the streamwise vortex cores, instead of being pinched into a triangular region immediately downstream of tube row i due to the presence of tubes in staggered row i + 1, actually diverge far enough to travel around the sides of the tubes in row i + 2, where their enhancement effects would be diffused over the [smaller] local heat transfer coefficients farther downstream. These and other assumptions are difficult to evaluate, especially when we are forced to use empirical correlations as a surrogate for the underlying data. Some of those correlations, such as Granryd's, are derived from data sets that included both inline and staggered tubes, thus making it impossible to test physical assumptions about the downstream path of streamwise vortices.

Other assumptions may underestimate the enhancement effects. For example the enhancement factor reported by Ichimiya was obtained by placing an acrylic cylinder on top of a heated metal plate and measuring local temperature profiles. Since the insulating effect of the cylinder would have produced unmeasurable variations in local heat flux, it may have caused the heat transfer coefficients calculated using local surface temperatures would represent an lower bound.

Recall from Fig. 2 that the enhanced area extended in a transverse direction about 0.35 tube diameters. Thus for heat exchangers having tube pitch less than 2.7 diameters, the enhanced areas may overlap laterally and invalidate the superposition model employed here.

At the current stage of development the semi-empirical correlation presented here is useful for exploring design tradeoffs in heat exchangers having large tube and fin pitch, as in many refrigeration applications where it is necessary to accommodate substantial frost accumulation, and defrost frequency may be constrained by marketing considerations. It may also apply to stationary applications where face area is constrained to the point where a deeper heat exchanger is needed to provide sufficient heat transfer area. In such cases, tube and fin pitch must be increased in order to minimize pressure drop, and thicker fins may be employed to offset the larger tube pitch.

The model's greatest value, after further validation, may be realized by using it to modify the form of the purely empirical expressions now used to predict air side performance of heat exchangers having smaller fin and tube pitches. By providing physically realistic asymptotes for such heat exchangers, and physically-based ways of dealing with the blockage effect of fin thickness near the leading edge and some developing flow effects, this semi-empirical approach may lead to development of a single expression that could extend throughout the parameter space. In particular, as more becomes known about the effect of fin pitch on the ability of von Karman vortex streets to form, and the rates at which both spanwise and streamwise vortices are dissipated, it may be possible to make the expressions more broadly applicable and even more physically realistic.

NOMENCLATURE

The nomenclature should be located at the end of the text using the following format:

Nu	Nusselt number	(-)
x*	normalized flow depth	(-)
D	tube diameter	(m)
EF	enhancement factor	(-)
N	number of rows	(-)
A	area	(m^2)
h	heat transfer coefficient	($W.m^2K$)
X	distance from tube surface	(m)
V	velocity	(m/s)

Subscripts

h	hydraulic
p	participants
loc	local
enh	enhanced
i	row number
D_h	hydraulic diameter
D	tube diameter

REFERENCES

Granryd, E., 1965, Varmeovergang och tryckfall vid patryckt stromning genom flanselement, *Kulde*, 19:4, p. 88-105

Ichimiya, Koichi, Norio Akino, Tomo Kunugi, and Koji Mitsushiro, 1988, Fundamental study of heat transfer and flow situation around a spacer (in the case of a cylindrical rod as a spacer) *Int. J. Heat Mass Transfer*. Vol. 31, No. 11, p. 2215-2225

Kakac, Sadik, Ramesh K Shah, Win Aung, 1987, *Handbook of single-phase convective heat transfer*, John Wiley & Sons, Inc.

Kawamura, Takao, Munehiko Hiwada, Toshiharu Hibino, Ikuo Mabuchi, and Masaya Kumada, 1984, Heat transfer from a finite circular cylinder on the flat plate. *Bulletin of JSME*, Vol. 27, No. 233

Kim, Yonghan and Kim, Yongchan, 2005, Heat transfer characteristics of flat plate finned-tube heat exchangers with large fin pitch. *International Journal of Refrigeration* Vol. 28 p. 851–858

Saboya F. E. M. and Sparrow E. M., 1974, Local and Average Transfer Coefficients for One-Row Plate and Tube Heat Exchangers Configurations, *Journal of Heat Transfer*, p. 265-272

Saboya F. E. M. and Sparrow E. M., 1976, Transfer characteristics of two-row plate fin and tube heat exchanger configurations", *Int. J. Heat Mass Transfer*, Vol 19 p. 41-49

Shah R. K. and London A. L., 1978, Laminar Flow Forced Convection in Ducts, *Supplement 1 to Advances in Heat Transfer*, Academic, New York

Stephan K., 1959, Warmeubergang und drauckabfall bei nicht ausgebildeter Laminarstromung in Rohren und in ebenen Spalten, *Chem.-Ing.-Tech.*, Vol 31, p. 773-778

Wang, C. C., Kuan-Yu Chi, and Chun-Jung Chang, Heat transfer and friction characteristics of plain fin-and-tube heat exchangers, part II: correlation, *Int. J. Heat & Mass Transfer*, Vol. 43, p. 2693-2700

Zhukauskas A. and Ambrazyavichyus A. B. 1972, "Heat Transfer from Tubes in Cross Flow" in J. P. Hartnett and T. F. Irvine, Jr., Eds., *Advances in Heat Transfer*, Vol. 8, Academic Press, New York

ACKNOWLEDGEMENT

This work was supported by the member companies of the Air Conditioning and Refrigeration Center at the University of Illinois at Urbana-Champaign.